Name: ______________________ **Math 254-2 Final Exam: 12/18/8** ID Code: ______________________

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Indicate clearly what work goes with which problem. Do not detach this first page. Cross out work you do not wish graded; incorrect work can lower your grade. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth 10 points; your total will be scaled to fit the standard 100 point scale. You have approximately 120 minutes.

1. Carefully define the term "vector space". Give two examples, neither of which may be $\mathbb{R}^n$ for any n.
2. Carefully define the term "independent". Give two examples in $\mathbb{R}^2$.

The following two questions involve the linear mapping $f : \mathbb{R}^3 \to \mathbb{R}^4$ given by $f(x, y, z) = (x - z, x - 2y, 2y - z, 6y - z - 2x)$.

3. Represent f as a matrix multiplication.
4. Determine the rank and nullity of f.

The following two questions involve the matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix}$.

5. Find A^{-1}, the inverse of A.
6. Use A^{-1} to find all solutions to the linear system $A[p\ q\ r]^T = [1\ 2\ 3]^T$.

7. Find all solutions to the linear system $\{u - v + w = 1,\ u + 2v - 4w = 3,\ 2u + v - 3w = 4,\ 3u - 2w = 5\}$.
8. Consider the linear system $\begin{pmatrix} 2 & 1 \\ 4 & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$. For which values of a does this system have no solutions? For which values of a does this system have one solution? Infinitely many solutions?
9. Set $U = \{(a, b, c) : ab + c = 6; a, b, c \text{ are real}\}$, a subset of $\mathbb{R}^3$. Give three vectors from U, and determine whether or not U is a vector space.
10. Set $V = \mathbb{R}^5$. Give any two subspaces U_1, U_2 such that $U_1 \oplus U_2 = V$.
11. Let W be the vector space of functions that have as a basis $B = \{1, e^{2x}, \sin x, \cos x\}$. Let L be the linear operator on W that acts via $L(f(x)) = f''(x) - f'(x) + 3f(x)$. Find the representation $[L]_B$.

The following two questions involve the vector space $\mathbb{R}^3$ under the standard inner product, and the subspaces $S = \text{span}\{(1, 1, 2), (2, -1, 3)\}$ and $T = \text{span}\{(1, -2, 1)\}$.

12. Find an orthogonal basis for S.
13. Determine the dimensions of each of $S, T, S + T, S \cap T$.

14. Find the determinant of $A = \begin{pmatrix} 1 & 1 & 0 & -1 \\ 0 & 2 & 3 & 1 \\ 2 & 1 & 0 & 2 \\ -1 & 2 & 1 & 0 \end{pmatrix}$.

The following two questions involve the matrix $A = \begin{pmatrix} -3 & 2 & 1 \\ -8 & 5 & 2 \\ 0 & 0 & 1 \end{pmatrix}$.

15. Find all of A's eigenvalues. Find a basis for each eigenspace.
16. For each eigenvalue, give the algebraic and geometric multiplicity. What is A's Jordan canonical form?

Please hand in all four pages. Do not detach this page.